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Transport and mixing in plasma turbulence.

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Transport and mixing in plasma turbulence

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Outline

**Motivation:** In magnetically confined plasmas turbulent – anomalous – transport is the dominant mechanism for transport of mass, energy and momentum! ([Balescu, *Aspects of Anomalous Transport in Plasmas*, CRC Press 2005])

- Turbulent dispersion and mixing of (passive) particles:
  - in drift wave turbulence
  - in global interchange turbulence in the edge/scrape-off-layer, SOL

- Relation between the passive particle diffusion and bulk plasma transport – the turbulent density flux

- Passive tracer particles/fields are used to model impurity transport

- Inertia effects; pinching and clustering in 2D drift wave turbulence

- Particle mixing and transport in strongly intermittent turbulence in SOL: Curvature pinch in inhomogeneous magnetic fields.
Particle dispersion in plasma turbulence

Vorticity

Drift-wave turbulence
Hasegawa Wakatani Equations
PRL 50, 682 (1983)
Particles convected by fluctuating $E \times B$ velocity

Basu et al Phys. Plasmas 10, 2696 (2003);
Dynamics of passive tracers

- Investigation of mixing and diffusion properties of turbulence – provides a diffusion coefficient – turbulent flux via Fick’s law.
- Passive tracers or passive fields are also widely applied for modelling impurity ion transport in plasmas: impurities are all materials besides the bulk plasma species; but here is does not include dust particles, only impurity ions are treated!
- Impurities, e.g., originate from sputtering off plasma facing components, PFC
- Passive tracers/fields do not contribute to charge neutrality and do not dynamically react back on the turbulence
- Severe condition: impurity density should be much smaller than density of bulk ions and contribution to quasi-neutrality condition much smaller than each of the contributing terms. (Naulin et al Phys. Scripta T122, 129 (2006)).

Passive scalar dynamics is a classical problem in fluid dynamics
Falkovich et al. Rev. Mod. Phys. 73, 913 (2001);
Tracing particles in the turbulence

Up to 100,000 particles are adverted in resistive drift-wave turbulence – Hasegawa-Wakatani (PRL 1983) model 2D.

\[ \vec{x}(t) = \vec{x}_0 + \int_0^t \vec{v}(\vec{x}(t'), t')dt \]

Principal component of \( \vec{v} = (u, v) \) is the \( E \times B \)-velocity, \( \vec{v}_E \): Ideal inertia-less particles.

Inertial effects: adding the polarization drift, \( \vec{v}_p \)

\[ \vec{v}_p = -\zeta \left( \frac{\partial}{\partial t} + (\vec{v}_E \cdot \nabla) \right) \nabla \varphi \]

\[ \zeta = \frac{eM}{qm_i} \frac{\rho_p}{L_n} \]

Important for heavier impurities!

Inertia effects make the advection velocity compressible!
The mean square particle displacement radially and poloidally.

- $x$ - radial direction,
- $y$ – poloidal direction

The radial particle displacement.

Fit: $At^\beta + B$ for $t > 400$; $\beta = 1$.

Asymptotically normal diffusion!

Diffusion coefficient $D_x = \langle X^2(t) \rangle / 2t$

Particle trapping in moving vortical structures:
Diffusion coefficient and flux

Particle density flux:
\[ \Gamma = n v_x = n v_{\text{ExB}} \]

Fick’s law:
\[ \Gamma = - D d_x n_0 \]

Comparison between \( D_x \) for tracer particles and \( D \) from the flux \( \Gamma \);
\[ D = - \Gamma / d_x n_0 \]

For the present case – HWe, fluctuations around a frozen background profile - passive particle diffusion really mimics bulk plasma transport!
Evolution of impurities as a passive field

The impurities are treated as a passive scalar advected by the turbulent fluctuations, i.e., the impurities do not act back on the turbulence or the background plasma profile.

\[ \partial_t n_{imp} + \nabla \cdot (\vec{v} n_{imp}) = \mu \nabla^2 n_{imp} \]

The influence of inertia enters via the polarization.

\[ (\partial_t + \vec{v}_E \cdot \nabla) n_{imp} = \zeta \nabla \cdot (n_{imp} (\partial_t + \vec{v}_E \cdot \nabla) \nabla \phi) + \mu \nabla^2 n_{imp}. \]

Restriction \( n_{imp} \ll n! \)

Lagrangian invariant: \( (\partial_t + \vec{v}_E \cdot \nabla)(\ln n_{imp} - \zeta \omega) \approx 0 \)

Turbulent mixing will homogenize the Lagrangian invariant:
\[ \ln n_{imp} - \zeta \omega \approx \text{const}. \]

The initially homogeneous impurity density field will granulate.
Clustering/aggregation of inertial impurities

The impurity equation may be written as:

$$D_t (\ln n_{imp} - \zeta \omega) = o(\zeta^2)$$

$$n_{imp}/n_{imp0} \sim \zeta \omega$$

Positive impurities ($\zeta > 0$) (this case) cluster in positive vortices

Negative impurities ($\zeta > 0$) will cluster in negative vortices

Impurity density and vorticity

Scatter plot of impurity density and vorticity, $\zeta = 0.05$, $\zeta = 0.01$, and $\zeta = 0.002$.

Linear regression: $\theta/\theta_0 = 1 + K\omega$; $K = 0.82\zeta$ ($\theta \equiv n_{imp}$)

Impurity pinch

Finite inertia also introduce a pinch effect: the (positive) impurities are transported up the density gradient – negative pinch velocity

Specific properties of the HWe ?
Turbulence and transport in the edge/SOL

In the edge/scrape-off-layer (SOL) region turbulence and transport is strongly intermittent and characterized by:

- large-amplitude, radially propagating blob-like structures of particles and heat, generated close to the last closed flux surface (LCFS),
- resulting in asymmetric conditional wave forms, and skewed and flattened PDFs with broad tails
- results in localized power loads at PFCs.

Observed under a variety of conditions (linear to toroidal devices):
Blob propagation in Alcator, C-Mod

Observations of density blobs at the outboard midplane of Alcator C-Mod (D^® - Light) O. Grulke et al. POP 13, 012306 (2006).


Inferred observations in the poloidal/toroidal direction

Magnetic field line

Radial velocity: 0.05 – 0.1 of the sound speed
Simulations of Edge-SOL

Risø ESEL code: interchange dynamics at the outboard midplane of a toroidally magnetized plasma. B-field gradient and curvature. Global evolution.

\[ S \propto \exp\left(-\frac{x^2}{2\delta_0^2}\right) \quad \sigma \propto 1 + \frac{1}{2}\tanh\left(\frac{x - x_l}{\delta_l}\right) \]

Energetics and energy transfer

Bursting: Kinetic energy contained by the mean, $U$, and fluctuating, $K$, motions.
The collective energy transfer terms $F_p$ and $F_v$. 
Spatial structure during a burst

Formation and propagation of density blob.
Particle density (left) and vorticity (right) during a burst ($\Delta t = 500$), radial blob velocity $< 0.02c_s$. 
Re-scaled PDF of particle density flux, $\Gamma = (n - \bar{n}) v_x$, measured at the probes, $P_i$.

Exponential tails: flux dominated by strong bursts.
Dynamics of impurity ions

The passive tracer particles model impurity dynamics, in the limit of no back-reaction on the plasma dynamics:

**Impurity density is much lower than the plasma particle density.**

(Naulin PRE ’05; Priego *et al* PoP ’05; Naulin *et al* Physica Scripta ’06)

Particles are advected as:

\[
\frac{d\vec{x}}{dt} = \vec{v} = \frac{1}{B(x)} \hat{z} \times \nabla \phi
\]

Finite inertia effects are neglected; \( \vec{v} \) is compressible due to the spatial dependence of \( B(x) \)

Garcia *et al* EPS 2005
Particle dynamics

Trajectory of a test particle released inside LCFS

Variogram of the particle motion,
- $\tau^2$;
- $\tau$;
- $\tau^{1.4}$
PDF of the radial displacement, $\Delta x$, over $\Delta t = 50$; all particles. $\langle \Delta x \rangle = -0.08$, standard deviation, $\sigma = 1.02$, skewness, $S = 0.4$, and kurtosis, $K = 10.7$. 

**Broad exponentially decaying tails.**

Long steps are almost equally probable in both in- and outgoing directions.
Particle dispersion

Particles released at $39 < x < 41$
Evolution of the impurity density

Released in $39 < x < 41$

Released in $159 < x < 161$

Evolution of the impurity/tracer particle density $N_0$ averaged over $y$. 
Arrival times

Plane at $x = 80$

Plane at $x = 160$

The relative number of particles passing through a radial plane versus time; first passage. Particles released inside LCFS, $39 < x < 41$.

Velocity of the front of the particles $> 0.02c_s$, typical blob speed.
Evolution of the impurity density

Density profile $N_0(x) \propto B(x)$ independent of release position. The transport is not “Fickian” diffusion. It can be described by an effective pinch:

$$\left( \frac{\partial}{\partial t} + \frac{1}{B} \hat{z} \times \nabla \phi \cdot \nabla \right) \frac{N}{B} = 0,$$

$N/B$ is a Lagrangian invariant: Effective turbulent mixing: $N/B$ uniformly distributed in space.

Impurities are effectively mixed by the turbulence in the SOL within a few burst periods. Even if originating far out in the SOL they will quickly penetrate across the LCFS into the edge plasma. Corresponding to the so-called inward (curvature) pinch.
Summary

- Dynamics of passive particles in turbulence:
- Diffusion coefficient mimics bulk plasma transport for a “fluctuation” model
- Modelling impurity transport by passive particles:
- Clustering/aggregation of inertial impurities and “inertial pinch”
- Edge/SOL turbulence and transport in a magnetically confined plasmas is bursty/intermittent with broad tailed PDFs and is not diffusive in the Fickian sense.
- No parametrized diffusion type equation: Transport characterisation calls for a universal PDF
- Impurities are effectively mixed in SOL and penetrates the LCFS.
- Impurity pinch: curvature pinch
Classical Particle Dispersion

Single particle dispersion: G.I. Taylor 1915
Stationary, homogeneous turbulent flows:

\[ R^2(t) = \langle (\vec{r}(t) - \vec{r}(t_0))^2 \rangle \]
\[ R^2(t) = 2\langle v^2 \rangle \int_0^t (t - \tau) C_L(\tau) d\tau \]
Lagrangian integral time scale; \( \tau_L = \int_0^\infty C_L(\tau) d\tau \)
\[ C_L(t) = \langle \vec{v}(\tau + t) \cdot \vec{v}(\tau) \rangle / \langle \vec{v}^2(\tau) \rangle \]

Two limits:
\[ t \ll \tau_L : R^2(t) = \langle v^2 \rangle t^2 \]
\[ t \gg \tau_L : R^2(t) = 2Dt \]
Diffusion coefficient: \( D = \langle v^2 \rangle \tau_L \)
General: \( R^2(t) \propto t^\alpha \)
\( \alpha > 1 \) Superdiffusion \( \alpha < 1 \) Subdiffusion

Fick's law:
\[ D^{\text{eff}} = \Gamma_0 / \nabla n_0, \text{ with normalizations } D^{\text{eff}} = \Gamma_0. \]
Particle dispersion in drift wave turbulence

Particle dispersion in 2D drift wave turbulence
Hasegawa-Wakatani equations (HWE): the resistive drift wave instability
(PRL 50, 682 (1983)):

\[
\begin{align*}
\partial_t n + \partial_y \varphi + \{\varphi, n\} &= -C (n - \varphi) + \mu_n \nabla^2 n \\
\partial_t \nabla^2 \varphi + \{\varphi, \nabla^2 \varphi\} &= -C (n - \varphi) + \mu_\varphi \nabla^4 \varphi
\end{align*}
\]

\[
1/C = 1/k_L^2 L || L || = (L_n T_e/m_e c_s \nu e_i)^{1/2}
\]

\[
\{\varphi, \psi\} \equiv \hat{z} \times \nabla \varphi \cdot \nabla \psi = \frac{\partial \varphi}{\partial x} \frac{\partial \psi}{\partial y} - \frac{\partial \varphi}{\partial y} \frac{\partial \psi}{\partial x}
\]

\[
u = -\frac{\partial \varphi}{\partial y}; \quad \psi = \frac{\partial \varphi}{\partial x}
\]

Normalization: \( \rho_s = c_s/\Omega_i \) for lengths; \( L_n/c_s \) for the times;
\( c_s = \sqrt{T_e/m_i}; \quad L_n = \left| (\nabla n_0(x)/n_0(x))^{-1} \right| \)
\( (T_e/e) (\rho_s/L_n) \) for potential \( n_0 \rho_s/L_n \) for density; \( \mu_n = \mu_\varphi = \mu \).

Basu et al Phys. Plasmas 10, 2696 (2003);
Energy spectrum

$k_x$ and $k_y$ spectra for $E = 2.0$, $n = 256$

Spectrum is isotropic
Running diffusion coefficient: $D_x = \frac{\langle X^2(t) \rangle}{2t}$, for varying adiabaticity parameter $C$. 
Turbulent particle density flux

Particle density flux:
\[ \Gamma = n v_x = n v_{\text{ExB}} \]

Bursty flux!

\[ p_G = \frac{1}{\pi} \frac{\sqrt{1-\gamma^2}}{\sigma_n \sigma_{v_x}} K_0 \left( \frac{|\Gamma|}{\sigma_n \sigma_{v_x}} \right) \exp \left( -\gamma \frac{\Gamma}{\sigma_n \sigma_{v_x}} \right) \]

\( \gamma \) is the correlation: \( \gamma = -\frac{\langle v_x n \rangle}{\langle v_x^2 \rangle^{1/2} \langle n^2 \rangle^{1/2}} = \cos \alpha \).
Turbulent particle density flux

Particle density flux: flux surface averaged

The probability distribution function for the plasma flux across the magnetic field is strongly non-Gaussian, i.e., strong bursts are dominating!
Edge/SOL turbulence transport in JET

Large intermittent burst

Radial velocity PDF

Re-scaled PDF of the radial particle velocity coarse grained over time intervals $\Delta t = 50 \cdot 2^{m-1}$; particles released inside LCFS

Re-scaled PDF of the turbulent radial ExB-velocity recorded at the probes $P_1 - P_7$