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Ultra-cold and Rydberg plasmas

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Ultra-cold and Rydberg plasmas

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Plasma effects in ultra-cold matter

Neutral gas in a MOT

$T \sim \mu K$

Equivalent to a non-neutral plasma

Rydberg plasmas

$T_i \sim mK, T_e \sim K$

Electron-ion "neutral" plasma

Bose-Einsten Condensates

$T \sim nK$

Plasma-like processes

Outline:

A - Plasma effects with neutral atoms

- Laser cooling forces;
- Hybrid mode: sound waves with a cut-off;
- Tonks-Dattner resonances;
- Density correlations.

B - Rydberg plasmas

- New dispersion relations;
- Magnetic field generation.

C - Bose Einstein Condensates

- Landau damping of Bogoliubov oscillations;
- Wakefield excitation;
- Two-stream instabilities.

Laser cooling force

Energy picture

|b>

$\hbar\omega_L < (E_b - E_a)$

Spontaneous emission

Absorption allowed by Doppler shift

|a>

The atom looses kinetic energy at each absorption-emission cycle

Momentum picture

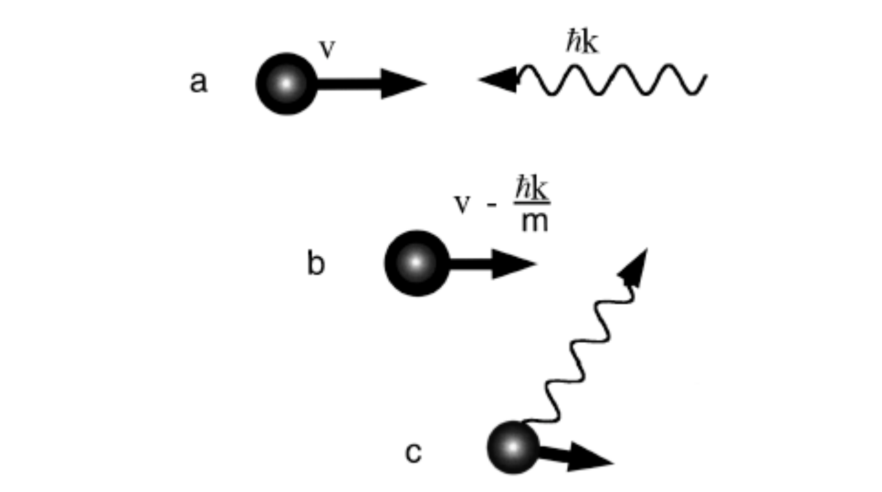


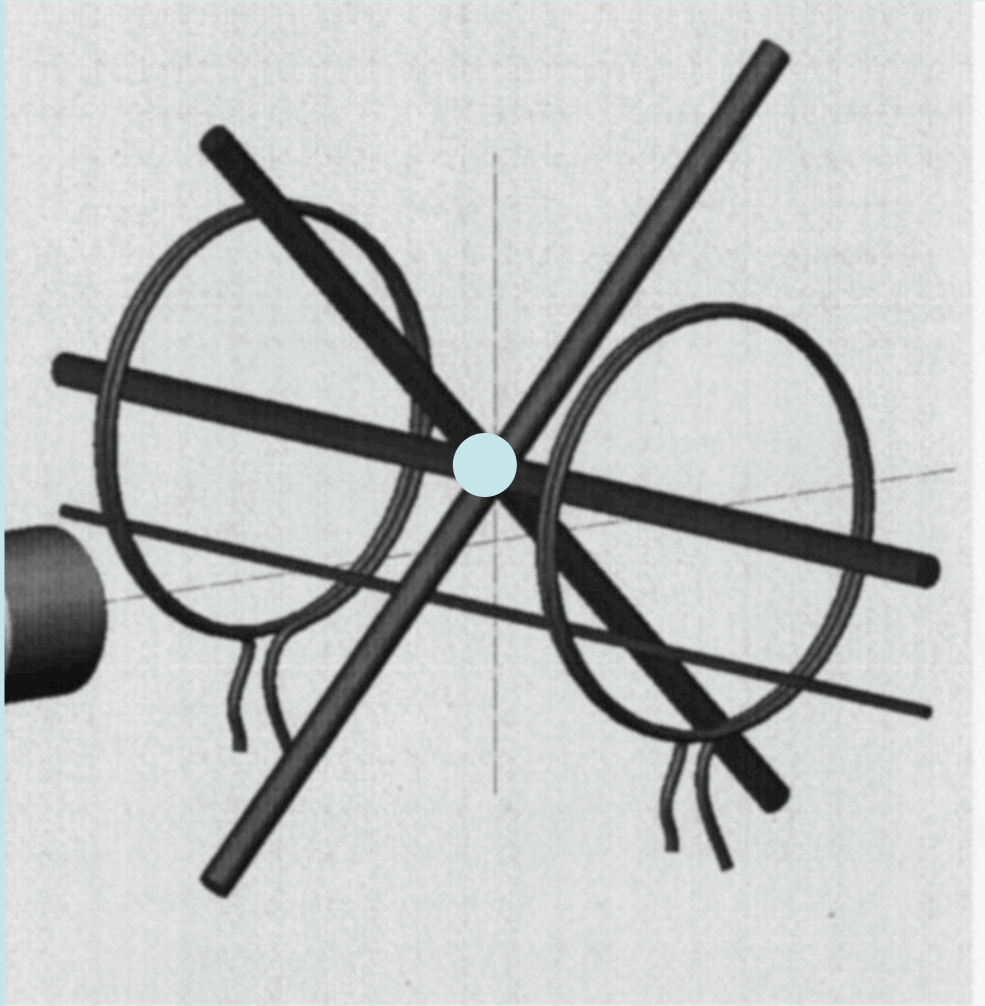
FIG. 1. (a) An atom with velocity v encounters a photon with momentum $\hbar k = h/\lambda$; (b) after absorbing the photon, the atom is slowed by $\hbar k/m$; (c) after re-radiation in a random direction, on average the atom is slower than in (a).

Taken from W.D. Phillips, RMP (1998)

1) Induced light pressure force [Ashkin, PRL (1970)]

$$F = F_0 - \beta v + O(v^2)$$

Magneto-optical traps (MOTs)



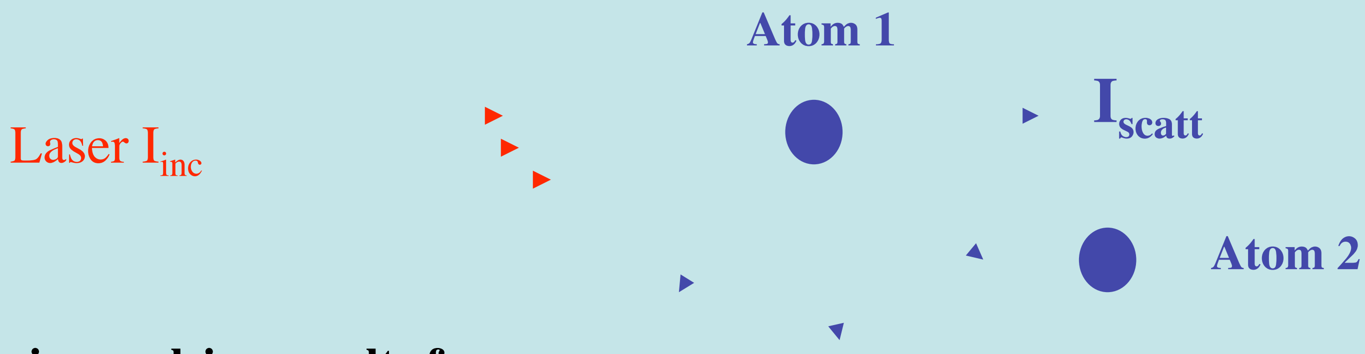
3 pairs of laser beams, for cooling

Helmotz coils, for magnetic confinement

Rubidium, the most popular cold gas

$5S_{1/2} \rightarrow 5P_{3/2}$ ^{85}Rb transition

Basic principle of the repulsive force



Atomic repulsion results from radiation pressure of the scattered radiation (I_{scatt} ~$1 / r^2$)

3) Shadow effect or absorption force [Dalibard, Opt.Commun. (1988)]

$$\vec{\nabla} \cdot [\vec{F}_A(\vec{r})] = -\sigma_L^2 \frac{I}{c} n(\vec{r})$$

2) Repulsive effect or radiation trapping force [Sesko et al., JOSA B (1990)]

$$\vec{\nabla} \cdot [\vec{F}_R(\vec{r})] = \sigma_R \sigma_L \frac{I}{c} n(\vec{r})$$

Collective forces in cold atom gas

Wave kinetic equation in the quasi-classical limit

$$\left[\frac{\partial}{\partial t} + \vec{v}\cdot\frac{\partial}{\partial \vec{r}} + \frac{1}{M}\left(\vec{F}_{conf} + \vec{F}\right)\cdot\frac{\partial}{\partial \vec{v}}\right]W = 0$$

Collective (shadow - repulsive) force

$$\nabla\cdot\vec{F} = Qn(\vec{r},t) \equiv Q\int W(\vec{v})d\vec{v}$$

Coulomb-like atom-atom interaction

$$Q = (\sigma_R - \sigma_L)\sigma_L I/c$$

Competing effect: repulsive force dominates over shadow effect

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Equilibrium

$$\vec{F}_{conf} + \vec{F}_0 = 0, \qquad \nabla \cdot \vec{F}_0 = Q n_0(\vec{r})$$

Perturbation

$$\delta\vec{F} = \vec{F}_{conf} + \vec{F} \propto \exp(i\vec{k}\cdot\vec{r} - i\omega t),$$

$$W(\vec{r},\vec{v},t) = W_0(\vec{v}) + \tilde{W}(\vec{v})\exp(i\vec{k}\cdot\vec{r} - i\omega t)$$

Linearized evolution equations

$$\tilde{W} = -\frac{i}{M}\frac{\delta\vec{F}\cdot\partial W_0/\partial\vec{v}}{(\omega - \vec{k}\cdot\vec{v})}$$

$$i\vec{k}\cdot\delta\vec{F} = Q\int \tilde{W}(\vec{v})d\vec{v}$$

Dispersion relation for cold atom gas (infinite geometry)

$$1 + \frac{Q}{Mk^2}\int \frac{\vec{k}\cdot\partial W_0/\partial\vec{v}}{(\omega - \vec{k}\cdot\vec{v})} d\vec{v} = 0$$

Dispersion relation similar to that of electrostatic waves in a plasma

$$1 + \chi(\omega, \vec{k}) = 0$$

Mono-kinetic distribution

$$W_0(\vec{v}) = N_0 \delta(\vec{v} - \vec{v}_0)$$

$$1 - \frac{QN_0}{M(\omega - \vec{k} \cdot \vec{v}_0)^2} = 0$$

For $v_0 = 0$, cold atom oscillations similar to plasma oscillations (compare with ω_{pe})

$$\omega = \omega_P \equiv \sqrt{\frac{QN_0}{M}}$$

Effective atomic charge

$$q_{eff} = \sqrt{\varepsilon_0 Q}$$

Typical experimental value, $q_{eff} = 10^{-6}$ e

Hybrid mode: sound wave with a cut-off

Fluid equations for the cold gas

$$\frac{\partial \vec{v}}{\partial t} + \vec{v}\cdot\nabla\vec{v} = -\frac{\nabla P}{Mn} + \frac{\vec{F}}{M}$$

$$\frac{\partial n}{\partial t} + \nabla\cdot(n\vec{v}) = 0$$

$$\nabla\cdot\vec{F} = Qn$$

$$P \propto n^{\gamma}$$

$$\frac{\partial^2 \tilde{n}}{\partial t^2} + \omega_P^2 \tilde{n} - u_s^2 \nabla^2 \tilde{n} = 0$$

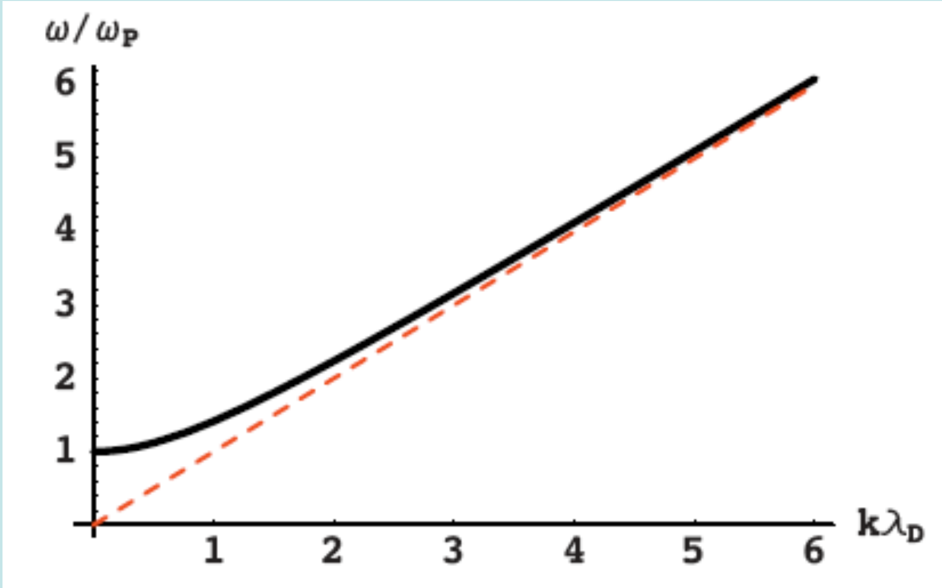
Dispersion relation

$$\omega^2 = \omega_P^2 + k^2 u_s^2$$

Sound speed

$$u_s^2 = \frac{5}{3}\frac{P_0}{Mn_0}$$

$$\lambda_D = u_s/\omega_P$$



Atomic Landau damping

Back to kinetics

$$\varepsilon(\omega,\vec{k}) \equiv 1 + \chi(\omega,\vec{k}) = 0$$

$$\chi_r(\omega,\vec{k}) = -\frac{1}{\omega^2}(\omega_P^2 + k^2 u_s^2)$$

$$\chi_i(\omega,\vec{k}) = i\pi \frac{Q}{Mk^2}\left(\frac{\partial W}{\partial v}\right)_{\omega/k}$$

W(v)

Non dissipative wave damping

$$\gamma = -\chi_i(\omega_r, k)/(\partial \chi_r / \partial \omega)_r$$

v

$$\gamma = \frac{\pi}{\omega}\frac{Q}{Mk^2}\left(\frac{\partial W}{\partial v}\right)_{\omega/k}$$

Diffusion in velocity space

Quasi-linear theory for a
broad spectrum of fluctuations

$$I(t) = \int I(\vec{k},t)\, d\vec{k} \Big/ (2\pi)^3$$

$$\frac{d}{dt} I(\vec{k},t) = 2\gamma_k(t) I(\vec{k},t) + S(\vec{k},t)$$

Diffusion equation

$$\left(\frac{\partial}{\partial t} + \vec{v} \cdot \nabla + \frac{\partial}{\partial \vec{v}} \cdot \overline{D} \cdot \frac{\partial}{\partial \vec{v}} \right) W_0(\vec{v},t) = 0$$

$$\overline{D} \propto \int I(\vec{k},t) \frac{\vec{k}\vec{k}}{(\omega - \vec{k} \cdot \vec{v})} \frac{d\vec{k}}{(2\pi)^3}$$

Fluctuations: an additional obstacle to atom cooling

Tonks-Dattner resonances

Internal oscillations in a Nonuniform cold gas

$$\nabla^2 \tilde{n} + k^2(\vec{r})\tilde{n} = \frac{\delta \vec{F}}{M u_S^2} \cdot \nabla n_0 + \frac{\nabla n_0}{n_0} \cdot \nabla \tilde{n},$$

$$k^2(\vec{r}) = [\omega^2 - \omega_P^2(\vec{r})] / u_S^2$$

a) Uniform slab

$$\frac{d^2 \tilde{n}}{dx^2} + \frac{1}{u_S^2}[\omega^2 - \omega_P^2(x)]\tilde{n} \approx 0$$

$$\omega_m^2 = \omega_P^2 \left[1 + \left(m + \frac{1}{2} \right)^2 \pi^2 \frac{\lambda_D^2}{L^2} \right]$$

m = 0, 1, 2, ...

b) Cylindrical geometry (plasma)

Parker, Nickel and Gould, PoP (1964)

c) Spherical geometry (neutral cold atom gas)

Mendonça et al., PRA (2008).

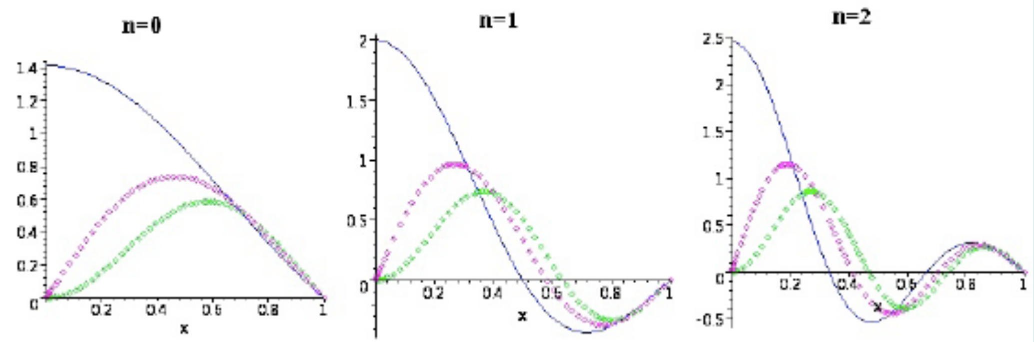


FIG. 2: *Profile of Tonks-Dattner modes, for $n = 0, 1, 2$ and $l = 0, 1, 2$.*

Centre of mass oscillations

Centre of mass position

$$\vec{R}(t)$$

$$\vec{R}(t) = \frac{1}{N}\int_V \vec{r}\,n(\vec{r})\,d\vec{r}$$

Neutral gas confined in a MOT

Dipole frequency for a single atom

$$\omega_D = \sqrt{K/M} \equiv \sqrt{Qn_0/M} = \omega_P$$

Similar to an electron-ion plasma

$$\frac{d^2\vec{R}}{dt^2} + \omega_M^2\vec{R} = \vec{f}(t)$$

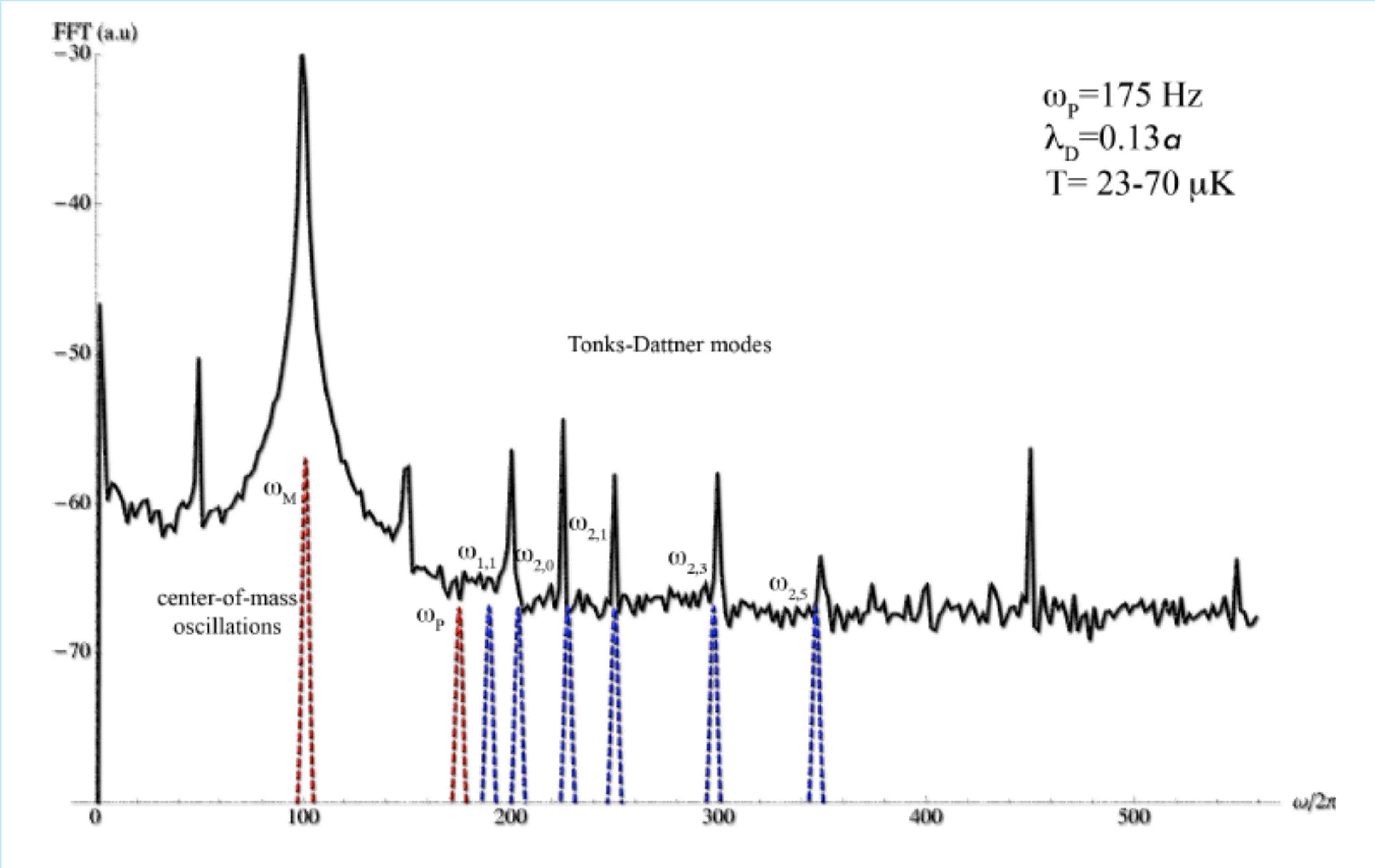
Mie frequency

$$\omega_M = \frac{Q}{M}\frac{1}{R^3}\int_0^R n(r)r^2dr$$

Constant density profile, $n(r) = n_0$

$$\omega_M = \sqrt{\frac{Qn_0}{3M}}$$

Experimental evidence of Tonks-Dattner resonances (to be confirmed)



Nonlinear coupling between dipole and plasma (or TD) oscillations

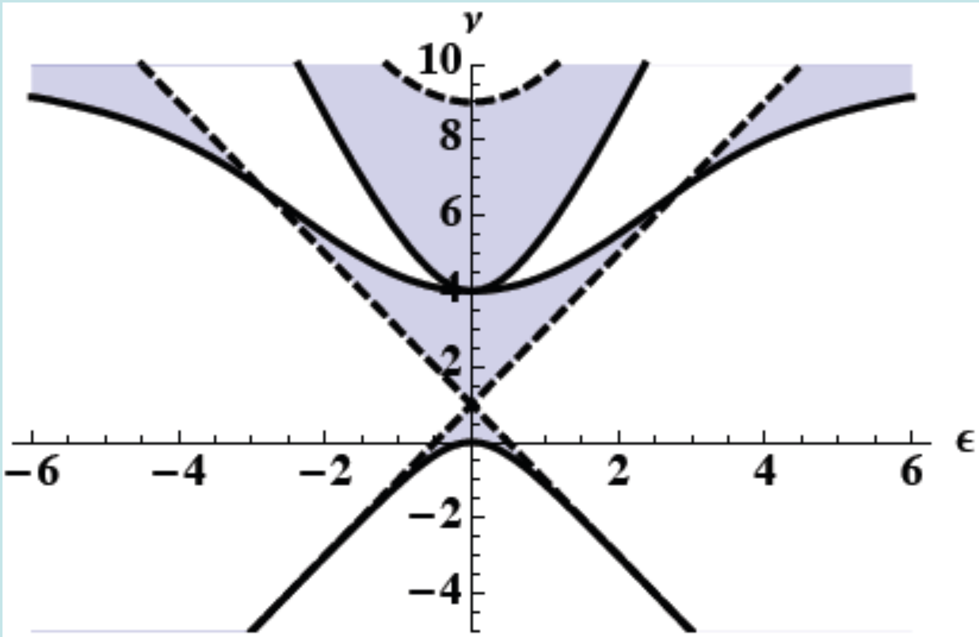
Density perturbations

$$\tilde{n}(\vec{r},t) = \tilde{A}(t)N(\vec{r})$$

$$\frac{\partial^2 \tilde{A}}{\partial \tau^2} + \left[\nu + 2\varepsilon\cos(2\tau)\right]\tilde{A} + 2\varepsilon\sin(2\tau)\frac{\partial \tilde{A}}{\partial \tau} = 0$$

Stability range

Mathieu-type of equation



Variables and parameters

$$2\tau = \omega_M t + \varphi$$

$$\nu = 4(\omega_P^2 + u_S^2 k^2)/\omega_D^2$$

$$\varepsilon = 2\vec{u}_0.\nabla \ln N(\vec{r})/\omega_D$$

Terças, Mendonça and Kaiser, PRA (sub 2009)

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Spectrum of density fluctuations

$$< |n(\omega', \vec{k}')|^2 >= n_0 \frac{F_0(\omega'/k')}{k'} |g(\vec{k}', \omega'/k')|^2$$

$$g(\vec{k}', \omega'/k') = \left[1 - \omega_p^2 \int \frac{(\partial F_0/\partial u) du}{k'^2 \epsilon(\vec{k}', \omega'/k')(u - \omega'/k')}\right]$$

Similar to that of a non-neutral plasma

Laser scattering

$\mathbf{E_0}$ $\mathbf{E_s}$

$$\omega = \omega_0 + \omega' \text{ and } \vec{k} = \vec{k}_0 + \vec{k}'$$

$$\frac{|E_s(\vec{k}, \omega)|^2}{|E_0|^2} = \frac{i\omega^4}{4k^2c^4} |\chi_a(\omega_0)|^2 (\vec{e}_\omega \cdot \vec{e}_0)^2 |n(\vec{k}', \omega')|^2$$

A possible (and practical) diagnostic technique

Mendonça + Terças, PRA (sub 2009)

Rydberg Plasmas

a. Creation of ultra-cold plasmas by photoionization of laser cooled Xe atoms [T.C. Killian et al., PRL (1999)]

b. Spontaneous evolution of a Rydberg cold Xe gas, into a plasma [M.P. Robinson et al., PRL (2000)]

Creation of ultra-cold plasmas (an apparent contradiction)

$$T_i \sim 100 \text{ mK}, T_e < \text{K}$$

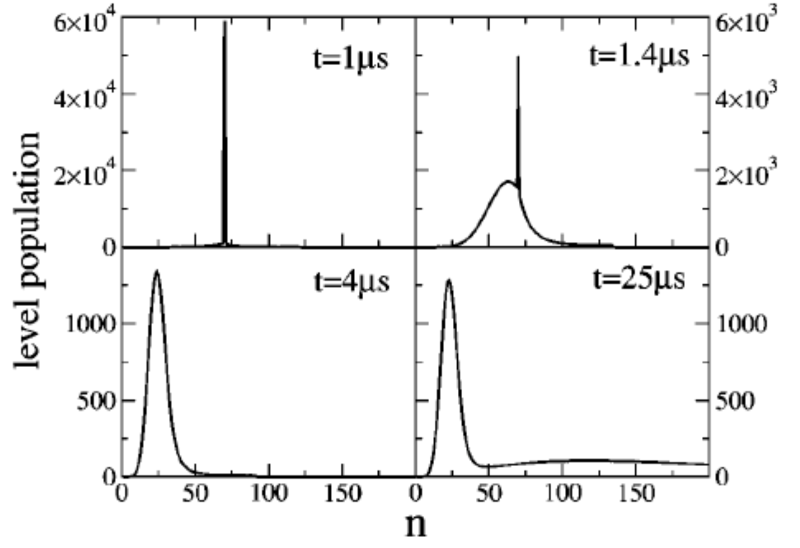


FIG. 3. Level distribution of Rydberg atoms after 1 μs, 1.4 μs, 4 μs, and 25 μs.

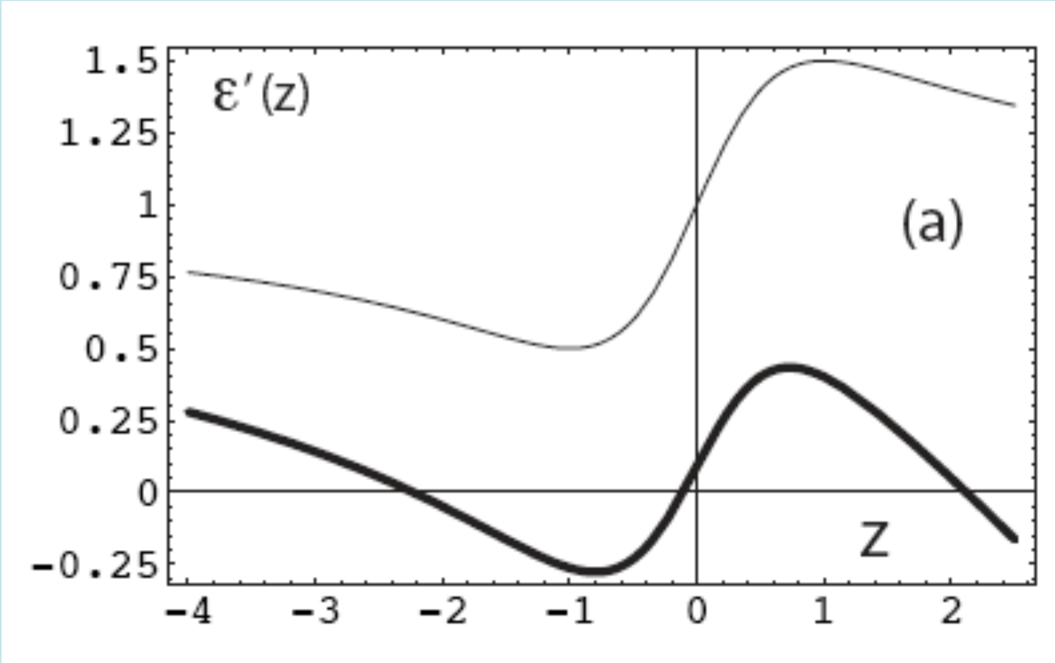
[T. Pohl et al. PRA (2003)]

Modified dispersion relation in a Rydberg plasma

$$\frac{k^2c^2}{\omega^2} = \epsilon(\omega) \equiv 1 + \chi_e(\omega) + N_a\chi_a(\omega)$$

Atomic susceptibility

$$\chi_a'(\omega) = -\frac{f_a}{n_0}\frac{\omega_{pe}^2\Delta}{(\Delta^2+\gamma^2)}D$$



$$z = \Delta/\gamma$$

Mendonça, Loureiro and Terças, JPP (2009)

Magnetic field generation in a Rydberg plasma

Ponderomotive force

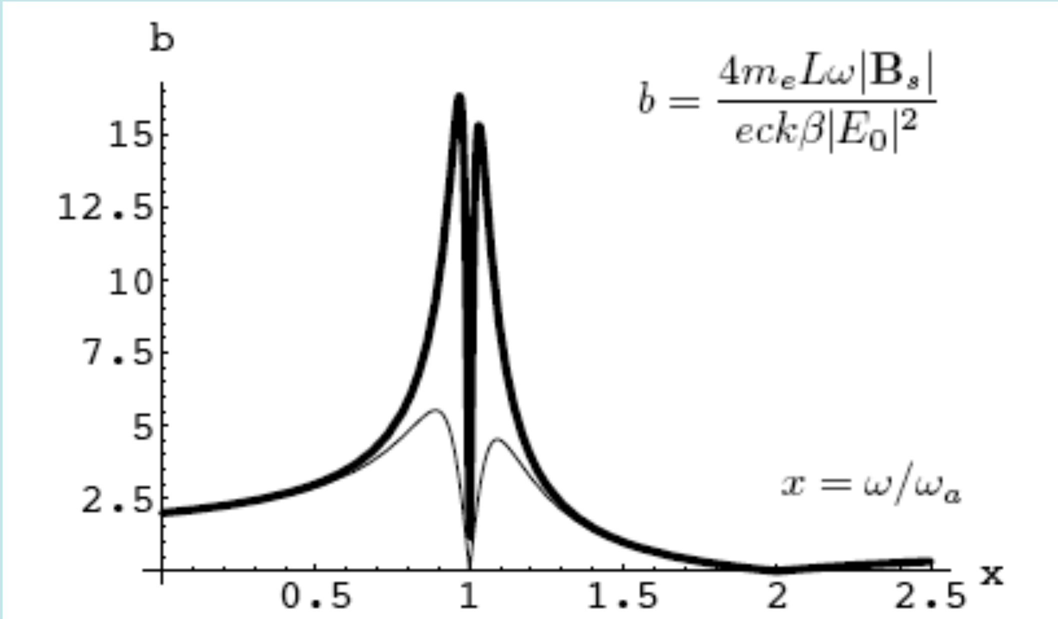
$$\mathbf{F}_p = \mathbf{F}_{ps} + \mathbf{F}_{pt}$$

$$\mathbf{F}_{ps} = \frac{(N-1)}{16\pi}\nabla|E_0|^2,$$

$$\mathbf{F}_{pt} = \frac{1}{16\pi}\frac{\mathbf{k}}{\omega^2}\frac{\partial[\omega^2(N-1)]}{\partial\omega}\frac{\partial|E_0|^2}{\partial t}$$

Quasi-static magnetic field

$$|\mathbf{B}_s| = \frac{eck\beta(2\omega_a-\omega)|E_0|^2}{4m_eL\omega(\omega-\omega_a)^2},$$



Enhancement around atomic resonance

Mendonça, N. Shukla and P. Shukla, JPP (2009)

Bogoliubov oscillations in a BE condensate

Exact kinetic dispersion relation

$$1 + \frac{q}{\hbar} \int W_0(k_z) \left[\frac{1}{(\Omega_+ - q v_z)} - \frac{1}{(\Omega_- - q v_z)} \right] \frac{dk_z}{2\pi} = 0$$

$$\Omega_\pm = \Omega \pm \frac{\hbar q^2}{2m}$$

Mono-energetic BEC beam $W_0(k_z) = 2\pi W_0 \delta(k_z - k_0)$

Dispersion relation for a cold beam

$$(\Omega - q v_0)^2 = q^2 C_s^2 + q^4 \frac{\hbar^2}{4m^2}$$

Bogoliubov sound speed

$$C_s = \sqrt{q W_0 / m}$$

Mendonça, Serbeto and Shukla, PLA (2008)

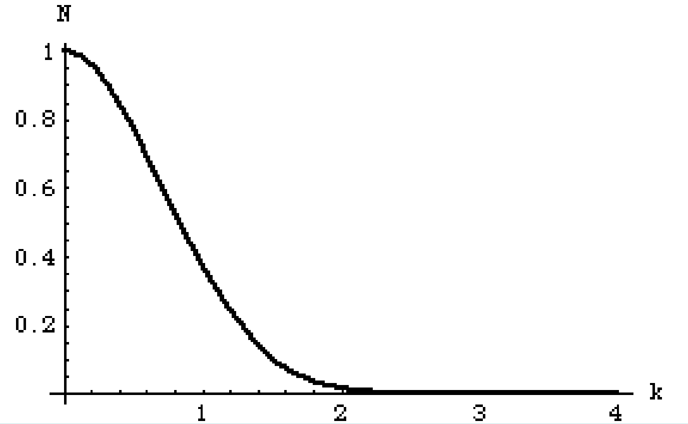
Landau damping of Bogoliubov oscillations

Exact quantum result, where atom recoil is included

$$\gamma = \frac{g^2 W_0}{4\hbar^2}\frac{q}{\Omega}\left[W_0(z=\Omega_+) - W_0(z=\Omega_-)\right]$$

Quasi-classical limit

$$\hbar q/m \ll \Omega/q.$$



Damping by resonant neutral atoms

$$\gamma = \frac{\omega}{4}\frac{gm}{\hbar^2}\left(\frac{\partial W_0}{\partial k'}\right)_{k'=k'_s}$$

$k'_s = mc_s/h$

BEC moving in a non-condensed gas

Unperturbed BE condensate beam N_0:

$$\left(\frac{\partial^2}{\partial t^2} - u_s^2 \nabla^2\right) \tilde{n} = 2n_0 \frac{g}{m} \nabla^2 N_0(x - u_0 t)$$

Sound velocity in the background gas, u_s

$$\tilde{n}(\eta) = 2n_0 \frac{g}{m} \frac{1}{(u_0^2 - u_s^2)} \left[N_0(\eta) - \int_\infty^\eta N_0(\eta') \sin(\eta - \eta') d\eta'\right]$$

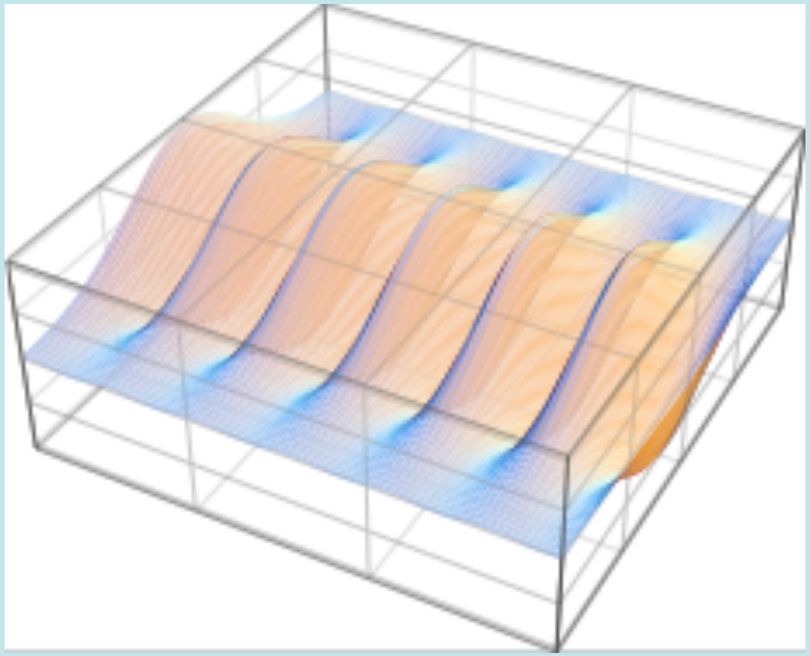
Wakefield solutions

Solution in the quasi-static approximation (no time variations in the moving frame)

- **First term: local perturbation of the background**
- **Second term:wake oscillation**

Wake frequency in the lab frame

$$\omega = \frac{(u_0^2 - u_s^2)^{1/2}}{u_0} \omega_0$$



Mendonça, Shukla and Bingham, PLA (2005)

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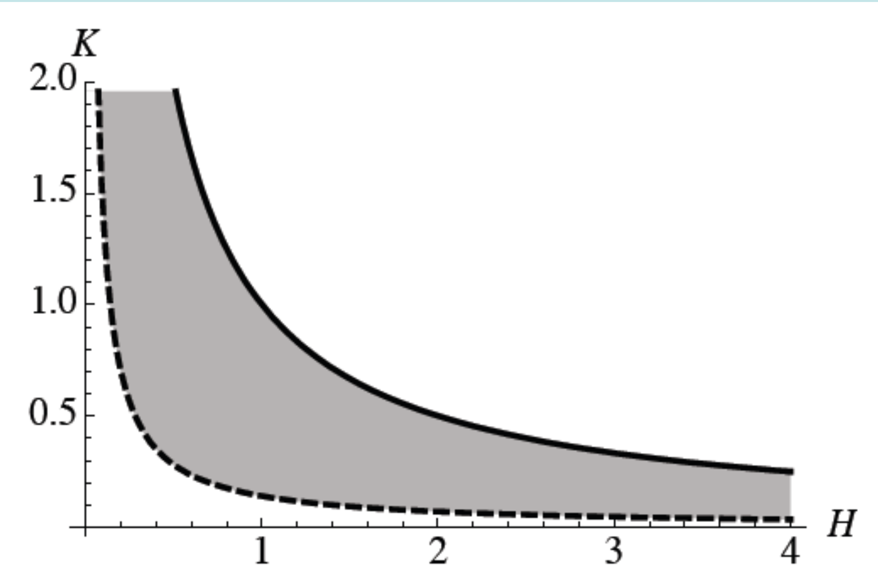
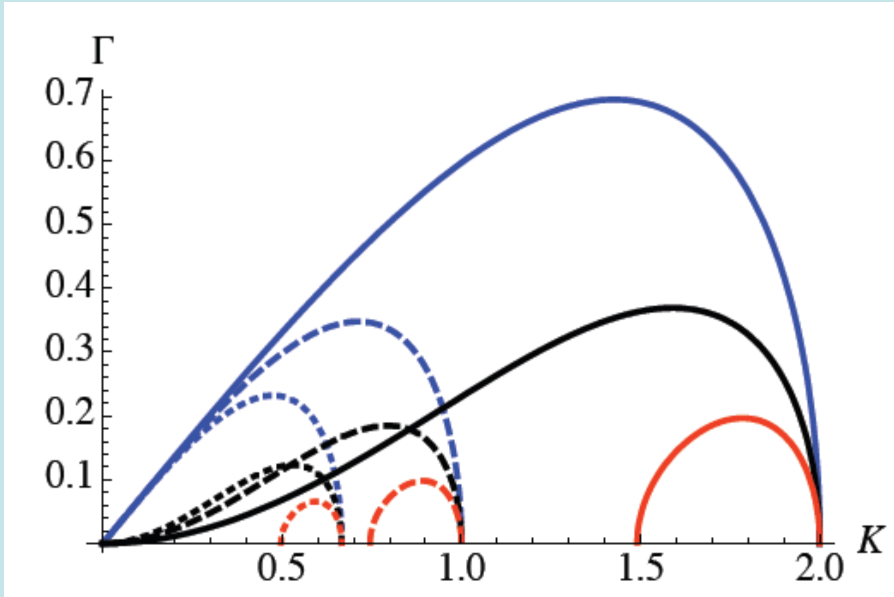
Two counter-streaming BECs

$$1 + \frac{g}{\hbar}\int \frac{dk}{2\pi} W_0(k)\left[\frac{1}{\Omega_+ - qv} - \frac{1}{\Omega_- - qv}\right] = 0,$$

Dispersion relation

$$K = qv_0/\omega_0, \qquad H = \hbar\omega_0/mv_0^2$$





H. Terças, JT. Mendonça and G. Robb, PRA (2009)

Conclusions

- Neutral ultra-cold gas in a MOT behaves like a plasma;
- Collective effects are due to shadow-repulsive forces;
- New hybrid modes (sound waves with a cut-off) were identified;
- Mie and Tonks-Dattner resonances: experimental evidence;
- Rydberg plasmas: modified dispersion relations, and B excitation;
- Bose Einstein Condensates (BECs) show plasma type of behavior;
- Quantum Landau damping of Bogoliubov oscillations;
- Two-stream instability of counter-streaming BECs.